2019 IEEE Intelligent Transportation Systems Conference (ITSC)
Auckland, NZ, October 27-30, 2019

Energy Saving in Metro Transit Substation through Train Trajectory Optimization

Bo Jin, Xiaoyun Feng, Pengfei Sun*, Mo Chen and Yang Chai

Abstract— **Regenerative braking technology has been widely used in metro train control. Improving the utilization of regenerative braking energy (RBE) can reduce the energy consumption of metro transit system. A novel train trajectory optimization method is proposed, in which control regimes coasting and maximum acceleration (CO-MA) are inserted according to the substation power flow. MA regime is inserted when the substation power is negative, which means there is REB can be used. CO regime is inserted to connect the trajectory and adjust the runtime. Improving the utilization of RBE according to the substation power flow makes the control optimization of trains independent, which can avoid the complex cooperation of multi-train control. A case study based on Guangzhou subway line is studied, and the results show that the proposed method can improve the utilization of RBE and reduce substation energy consumption efficiently.**

I. INTRODUCTION

Metro transit system is developing rapidly to meet the increasing passenger demand due to its high capacity, reliability and convenience. However, with rising energy prices and increasing metro transit system size, huge energy costs have become a nonnegligible challenge. In recent years, researchers and practitioners have paid attention to reducing the energy consumption of metro transit system, and a number of energy-efficient measures have been proposed [1]. Energy-efficient train control (EETC) is a hotspot due to its low investment cost and high energy saving potential, which focuses on train trajectory optimization. There are two comprehensive surveys about EETC [2] [3].

Reducing tractive energy consumption is the main optimal object of EETC. Based on Pontryagin maximum principle, Ishikawa [4] proposed that the optimal train control strategy consisted of maximum acceleration (MA), cruising (CR), coasting (CO) and maximum braking (MB). The analysis was based on a train journey between two stations on a level track. Pudney and Howlett [5] introduced conditions of varying gradients and speed limits into the problem. Howlett *et al.* [6] proved that the optimal train control strategy was suitable for the metro transit system. Methods in the literatures [4]-[6] belong to analytical algorithm, which can obtain the optimal solution exactly with complicated computational process. On the other hand, numerical algorithm is used to calculate energy-efficient train trajectory, there is a trade-off between solution accuracy and computational efficiency. Franke [7] taken kinetic energy as dynamic state variable to build a discrete position nonlinear model, in which dynamic programming was used to optimize train trajectory. Haahr [8] built a time-space graph by dividing the track into different phases according to running conditions, and solved the problem by dynamic programming. Jin *et al.* [9] transformed the nonlinear EETC problem into a mixed inter linear planning problem, which can be solved efficiently. However, the use of piecewise affine reduced the computational accuracy. According to the short distance between stations and simpler line condition, coast control [10] [11] is proposed to calculate energy-efficient train trajectory for metro transit system. The key to coast control is searching the starting point of CO regime. A genetic algorithm was proposed to optimize train trajectory using coast control, in which time points to initiate CO regime and resume MA regime was calculated [10]. Wong and Ho [11] focused on the multiple coasting point control, and they designed a hierarchical genetic algorithm to determine the number of coasting points.

Regenerative braking technology has been well applied in metro trains. Regenerative braking has a specific function that the braking kinetic energy can be recovered into electricity using an electric motor as an electric generator during the braking phase [12]. The feedback electrical energy can be consumed by other trains in the same substation, or wasted by heating resistors. Thus, improving the utilization of regenerative braking energy (RBE) can reduce substation energy consumption. Traditional reducing tractive energy consumption was not the same as reducing substation energy consumption, which ignored the utilization of RBE [13]. Miyatake and Ko [14] analyzed the relationship between train trajectory and state of charge. The simulation results showed that the regenerative braking played an important role in energy saving. Bocharnikov *et al*. [15] formulated a multi-train simulation to adjust the schedules and trajectories for both the up and down direction trains. The purpose was to reduce total energy consumption of a metro line by coordinating trains operation. Sun *et al*. [16] [17] developed a method to cooperate the operation of multi-train, in which train trajectory was partly adjusted to absorb the RBE. Cooperation of multi-train is complex task, because of its various cases and interaction of trains. On the other hand, timetable optimization is another method used to improve the utilization of RBE. Li and Lo [18] adjusted train trajectory and dwell time to minimize energy consumption for metro transit system operations. Zhou *et al*. [19] built an integrated model to reduce energy consumption, in which train trajectory and energy-efficient timetable were optimized respectively.

This work is supported in part by the Fundamental Research Funds for the Central Universities of China under Grant 2682017CX048 and in part by the collaborative project (Research on Energy-saving Optimization Control Technology for Subway Train Operation) with CRRC Qingdao Sifang Stock Research Institute Co., LTD.

The authors are with the School of Electrical engineering (Southwest Jiaotong University), Chengdu 611756, China.
*Corresponding author (e-mail: spf0325@163.com).

978-1-5386-7024-8/19/$31.00 ©2019 IEEE

This paper focuses on optimizing energy-efficient train trajectory according to the substation power flow. Same as previous work, train trajectory optimization is under the constraints of runtime, speed limits, line conditions and train conditions. Specially, in order to avoid the complex cooperation of multi-train in the same substation, power flow is introduced to show the energy influence of other trains. Meanwhile, according to the substation power flow, the train control regime CO-MA is inserted into the trajectory to absorb the RBE, in order to improve the utilization of RBE and reduce the system operation energy consumption.

The rest of this paper is organized as follows. In section II, the optimal model of reducing system energy consumption is stated. In section III, methods to calculate energy-efficient train trajectory are introduced. In section IV, case study of Guangzhou subway line is demonstrated to verify the feasibility of the model and method. Conclusions are given in section V.

II. MODEL STATEMENT

A. Notations

Notions used throughout the remainder of this paper are introduced as followings:

v	train speed;
x	train position;
t	train running time;
Δx	position interval;
m	train mass;
ρ	factor considering the rotating mass;
F_t	train tractive effort;
F_{eb}	train regenerative braking effort;
F_{ab}	train air braking effort;
W_0	basic resistance;
W_j	line resistance;
V_b	speed in which F_{eb} convert to F_{ab}, and vice versa;
S_k	position of the kth station;
T_k	runtime between the kth and k+1th station;
V	limited speed;
η	efficiency of electrical to mechanical conversion, and vice versa;
P_{sub}	substation electrical power;
P'_{sub}	original substation electrical power before the train departure;
P^+	positive substation electrical power;
P_t	electrical tractive power;
P_a	electrical auxiliary power;
P_b	electrical regenerative braking power;
P_{b_u}	utilized electrical regenerative braking power;
P_{loss}	substation electrical loss power;
$[T_s\ T_e]$	optimized time interval;

B. Metro transit system energy consumption

In order to measure energy efficiency, energy power flow of metro transit system is discussed in this section. Generally, energy consumption in metro transit system can be classified into two parts: non-traction and traction consumption. Non-traction consumption refers to the energy utilized for system operation expecting for train operation, like energy utilized at stations, depots and other facilities [1]. On the other hand, traction consumption accounts for the energy costed for train operation including train tractive energy consumption, auxiliary energy consumption, utilized RBE and energy losses. Traction consumption can be evaluated by the power of substation, which can be expressed as:

$$P_{sub} = P_t + P_a - P_{b\_u} + P_{loss} \tag{1}$$

For metro traction power system, as shown in Figure 1, trains detect electrical flow from substation to meet tractive energy consumption and auxiliary energy consumption. The electrical tractive power P_t can be calculated as:

$$P_t = F_t v / \eta \tag{2}$$

and, electrical auxiliary power P_a is decided by the auxiliary systems of the vehicle such as heating, ventilation, air-conditioning, lighting, which is assumed to be fixed in this paper. Electrical braking power P_b can calculated as:

$$P_b = \eta F_{eb} v \tag{3}$$

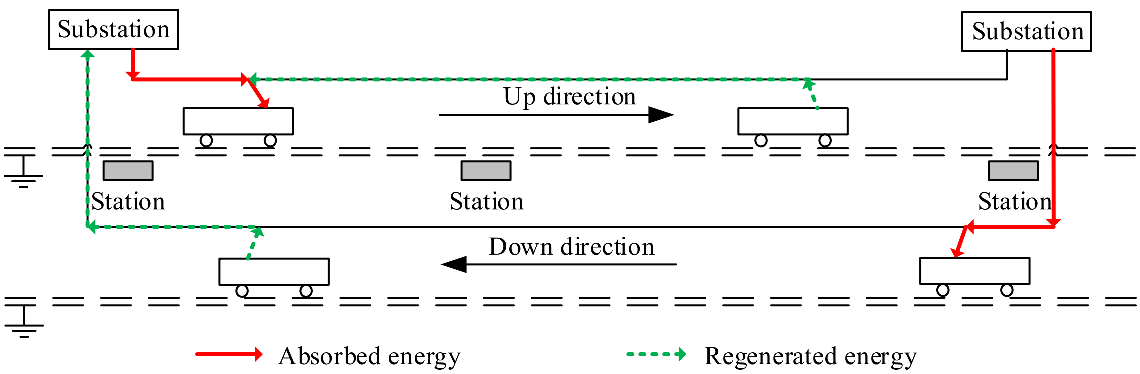

Figure 1. Metro traction power system with multi-train

which can feed back to traction power network and be utilized by other trains in the same substation (as shown in Figure 1) or wasted by heating resistors. The utilization rate of regenerative braking is influenced by the control regimes of trains in the same substation. Because of the high departure frequency and short distance between stations, frequent accelerating and braking make the energy-saving effect of using RBE is considerable. However, there is still much REB unutilized, which can be further used by adjusting train control regime. For example, setting the train control regime to MA to absorb the RBE. The energy storage technology is not considered in this paper. The calculating of substation electrical loss power P_{loss} is complex and not the focus of this paper, which is simplified to be fixed.

C. Metro train trajectory optimization model

The calculation of metro train trajectory is finished by the optimization layer of automatic train operation (ATO) system. The optimization layer optimizes the train trajectory according to line data, train data and objectives, as shown in Figure 2. Then, the control layer tracks the recommended trajectory based on the control strategy. Different with traditional optimization layer of ATO system, power flow is taken as input to influence the optimization of trajectory. Additionally, the substation power flow can be calculated according to the trajectory of trains in the same substation.

According to the novel optimization layer, the metro train trajectory optimization model is built. Firstly, the dynamics equation of train motion can be formulated as the following model:

$$\begin{cases} m\rho \cdot \dfrac{dv}{dt} = F_t - F_{eb} - F_{ab} - W_0 - W_j \\ \dfrac{dx}{dt} = v \end{cases} \tag{4}$$

The objectives of optimization layer consist of safety and punctuality. Then, constraints are built to make the train operation safely and on time, which are as followings:

$$\begin{cases} 0 \le v(x) \le V(x) \\ t(S_{k+1}) - t(S_k) = T_k \\ 0 \le F_t(v) \le F_{t,\max}(v) \\ 0 \le F_{eb}(v) \le F_{eb,\max}(v) \\ 0 \le F_{ab}(v) \le F_{ab,\max}(v) \\ F_t \cdot (F_{eb} + F_{ab}) = 0 \end{cases} \tag{5}$$

Another important objective of optimization layer is energy-saving, also the optimal objective of this paper, which can be expressed as:

$$\min \int_{T_s}^{T_e} P^{+}(t)dt \tag{6}$$

where, positive substation electrical power P^{+} can be calculated according to:

$$P_{sub} = P'_{sub} + P_t + P_a - P_{b\_u} + P_{loss} \tag{7}$$

$$P^{+}(t) = \begin{cases} P_{sub}(t) & if\ P_{sub}(t) > 0 \\ 0 & if\ P_{sub}(t) \le 0 \end{cases} \tag{8}$$

and the relationship between train effort and electrical power is show as Eq.(2) and Eq.(3).

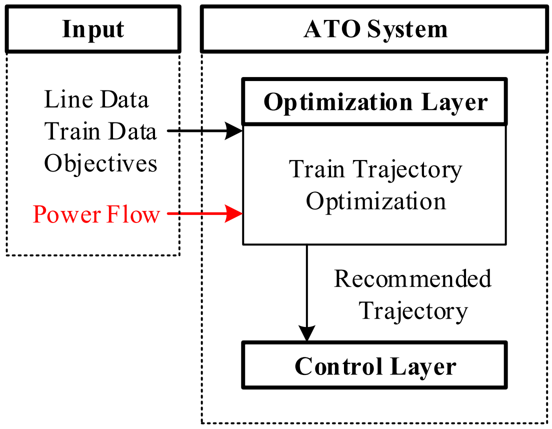

Figure 2. Structure of metro ATO system

III. METHOD

A traditional coast point searching method consists of two steps: calculating the MaxPow trajectory, inserting CO regime into the MaxPow trajectory to satisfy the runtime constraint. The MaxPow trajectory means that the train finishes the travel between two stations with minimum runtime. Different with traditional optimization method, substation power flow is known before the train departing in this paper. According to the character of power flow, MA regime is inserted to utilize the feedback RBE, as shown in Figure 3. Meanwhile, CO regime is inserted before the MA regime to connect it with the MaxPow trajectory. Thus, CO-MA regime is inserted into the MaxPow trajectory to improve the utilization of RBE and

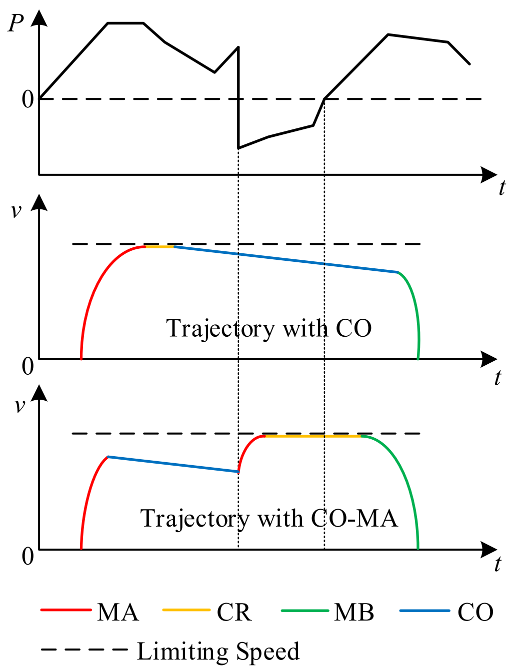

Figure 3. Electrical power and two kinds of trajectory

meet the runtime constraint. The calculation of MaxPow trajectory will be introduced firstly, and the calculation of trajectory with CO (TC) and trajectory with CO-MA (TCM) is introduced based on it respectively.

A. MaxPow trajectory

In order to achieve minimum runtime, the control strategy is combined of three regimes: MA, MB and CR, in order to get close to limited speed as soon as possible, as shown in Figure 4. The algorithm for calculating MaxPow trajectory can be summarized as followings:

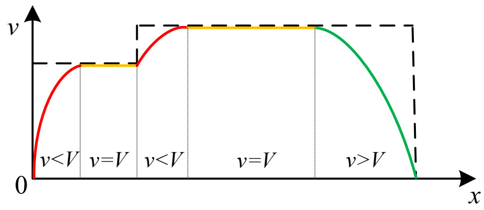

Figure 4. The MaxPow trajectory

Algorithm 1. Calculating MaxPow trajectory

1. Obtain line and train data, and initialize the state variable (v, F_t, F_{eb}, F_{ab}) of x at the departure station S_k.
2. Calculate state variables according to the relationship between speed $v(x)$ and limited speed $V(x)$, if $v < V$, go to step 3; else if $v = V$, go to step 4, else $v > V$, go to step 5.
3. Set train tractive effort $F_t = F_{t,\max}$ to achieve MA regime, then calculate the state variables of $x = x + \Delta x$ based on Eq.(4). If $x < S_{k+1}$, go to step 2, else end.
4. Set train tractive effort $F_t + F_{eb} + F_{ab} = W_0 + W_j$ to achieve CR regime, then calculate the state variables of $x = x + \Delta x$ based on Eq.(4). If $x < S_{k+1}$, go to step 2, else end.
5. Calculate the MB curve backward form position x, and set train braking effort, if $v > V_b$, $F_{eb} = F_{eb,\max}$ and $F_{ab} = 0$, else $F_{eb} = 0$ and $F_{ab} = F_{ab,\max}$. Connect the MB curve with precalculated trajectory, If $x < S_{k+1}$, go to step 2, else end.

B. Trajectory with CO

Based on the MaxPow trajectory, CO regime is inserted to reduce tractive energy consumption, as shown in Figure 5. The main task of calculating TC is searching the starting point of CO regime, which makes the trajectory satisfying the runtime constraint. The calculating algorithm is summarized as followings:

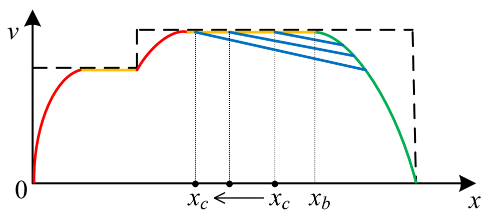

Figure 5. The trajectory with CO

Algorithm 2. Calculating trajectory with CO

1. Obtain the MaxPow trajectory and confirm the starting point of MB regime x_b.
2. Set the initial starting point of CO regime x_c, $x_c = x_b - \Delta x$.
3. Calculate the CO curve starting from x_c, in which train effort is set as $F_t + F_{eb} + F_{ab} = W_0 + W_j$ to achieve CO regime. Connect the CO curve with the MaxPow trajectory.
4. Calculate the runtime of the new trajectory, if it is equal to pregiven runtime, end, else set $x_c = x_c - \Delta x$ and go to step 3.

C. Trajectory with CO-MA

Based on the MaxPow trajectory and substation power flow, CO-MA regime is inserted to improve the utilization of REB, as shown in Figure 6. Firstly, the time interval in which the substation power is negative should be confirmed, which means there is REB can be used. Then, MA regime should be inserted to utilize it. However, if the control regime of the MaxPower trajectory in the time interval has already been MA, then there is no need to insert MA regime and TC is the energy-efficient trajectory. The calculating algorithm is summarized as followings:

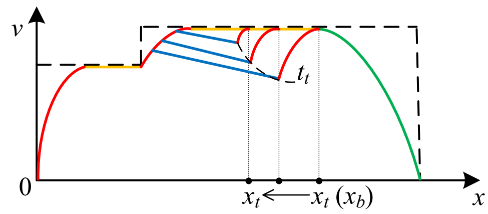

Figure 6. The trajectory with MA-CO

Algorithm 3. Calculating trajectory with CO-MA

1. Obtain the MaxPow trajectory and substation power flow.
2. Confirm the starting time t_t of the time interval in which substation power is negative, and confirm the starting point of MB regime x_b.
3. If the control regime of the MaxPow trajectory at t_t is MA, then go to Algorithm 2, else continue.
4. Initialize the end position of MA regime x_t, $x_t = x_b$.
5. Calculate the MA curve backward from x_t, make the starting time of the MA curve is equal to t_t.
6. Calculate the CO cure backward from the starting point of MA curve, and connect the CO curve with the MaxPow trajectory.
7. Calculate the runtime of the new trajectory, if it is equal to pregiven runtime, end, else set $x_t = x_t - \Delta x$ and go to step 5.

IV. CASE STUDY

In order to illustrate the model and methods proposed, case study is presented based on data from Guangzhou subway line. There are total 24 stations and 10 substations, and the substation consists of Station 1 to 3 is researched. The train running in the metro line, is A-type EMU with mass of 339.6t, type of 4M2T, factor to consider the rotating mass is 0.057, efficiency of electrical to mechanical conversion is 0.85, auxiliary power is 400W, and train efforts and basic resistance as Eq.(9)-(11). The timetable parameters of the substation between Station 1 to Station 3 and other simulation parameters are given Table I.

$$F_{t,\max}(v) = \begin{cases} 400 & 0 \le v \le 40 \\ 1600/v & 40 < v \le 55 \\ 870000/v^2 & 55 < v \le 80 \end{cases} \tag{9}$$

$$F_{eb,\max}(v) = \begin{cases} 0 & 0 \le v \le 5 \\ 389 & 5 < v \le 80 \end{cases} \tag{10}$$

$$W_0 = 9.067 + 0.001334v^2 \tag{11}$$

TABLE I. SIMULATION PARAMETERS

Inter-station length	**Length (m)**
between Station 1 and Station 2	1010
between Station 2 and Station 3	2400
Runtime	**Time (s)**
from Station 1 to Station 2	74
from Station 2 to Station 3	139
from Station 3 to Station 2	140
from Station 2 to Station 1	75
Dwell time	**Time (s)**
at Station 1	30
at Station 2	40
at Station 3	30
Headway	**Time (s)**
from Station 1 to Station 2	150
from Station 2 to Station 1	150

Two kinds of energy-saving trajectory, trajectory with CO (TC) and trajectory with CO-MA (TCM), are tested in virtual operational environment. The substation power flow of time interval [7:00 7:10] is shown in Figure 7. It is obvious that the positive power (which reflects the substation energy consumption) of TC is larger than the positive power of TCM, and the negative power (which reflects the wasted RBE) of TC is also bigger than the negative power of TCM. Thus, the application of TCM can reduce the substation energy consumption efficiently and wasted RBE. In order to further state that the application of TCM can improve the utilization of RBE, the operation of Train 1401 is discussed in more detail. Train 1401 departs from the Station 3 at 7:02:30 and arrives at the Station 2 at 7:04:09. The TC and TCM of Train 1401 are shown in Figure 8, the control regime of TCM in the time interval [t_1 t_2] is CO, and in the time interval [t_2 t_3] is MA. The power of TC and TCM are different because of different control strategies. The power flow in time interval [7:02:30 7:04:09] is shown in Figure 9. The substation power of TC in the time interval [t_2 t_3] is negative, which means there is REB unutilized. On the contrary, the power of Train 1401 is big enough to utilize the RBE fully, and the substation power of TCM in the time interval [t_2 t_3] is positive, which means the REB is this interval is utilized. From the perspective of system energy-saving, in the time interval [7:02:30 7:04:09], the substation energy consumption of TC is 124.40kWh, and of TCM is 123.23kWh, which is reduced 0.94%. And, the unutilized RBE of TC is 28.95kWh, and of TCM is 26.71kWh, the wasted REB is reduced 7.74%. From the case study, it can be seen that the application of TCM can improve the utilization of REB and reduce the substation energy consumption.

V. CONCLUSION

For energy-saving in metro substation, a novel train trajectory optimization method is proposed to improve the utilization of RBE. Different with traditional trajectory optimization method, substation power flow is taken as input to calculate more energy-efficient trajectory. Getting the unutilized RBE information from the substation power flow, MA regime is inserted when the substation power is negative, which makes the RBE can be utilized instead of wasted. CO regime in inserted before the MA regime to connect the trajectory and adjust the runtime. Meanwhile, the substation

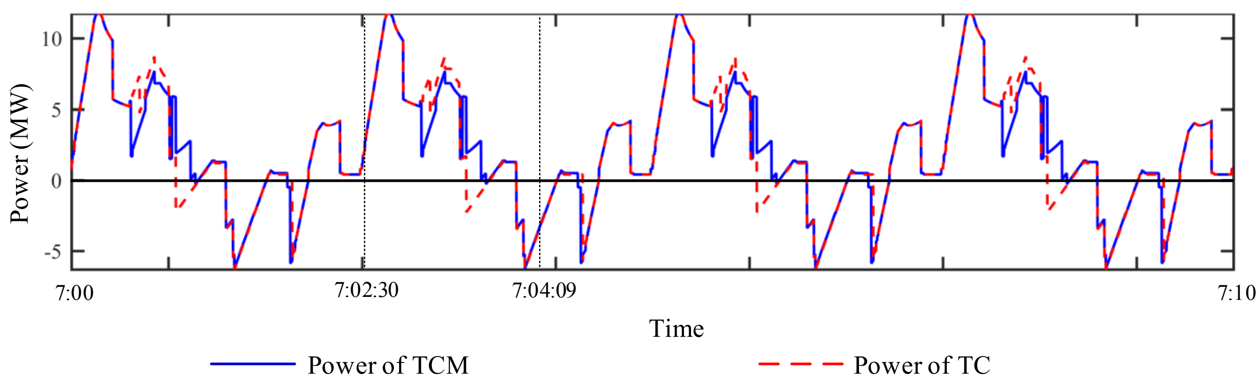

Figure 7. Subation power flow

power flow reflects the energy effect of other trains in the same substation, which can avoid the complex multi-train cooperation. A case study is presented based on Guangzhou subway line, which shows the TCM performances better in utilizing RBE and reducing substation energy consumption. As further work, control regime CO-MA should be inserted more considerately, the train effort of MA regime should be calculated according to the value of REB, in order to absorb RBE fully and avoid absorbing extra energy from substation. Additionally, the optimization of trajectory and timetable should be considered integrally.

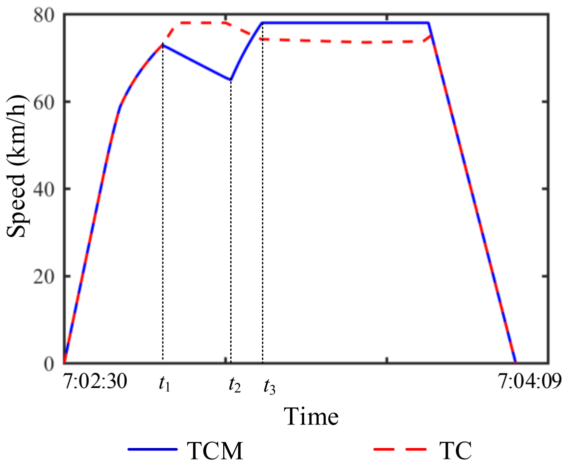

Figure 8. The trajectory of Train 1401

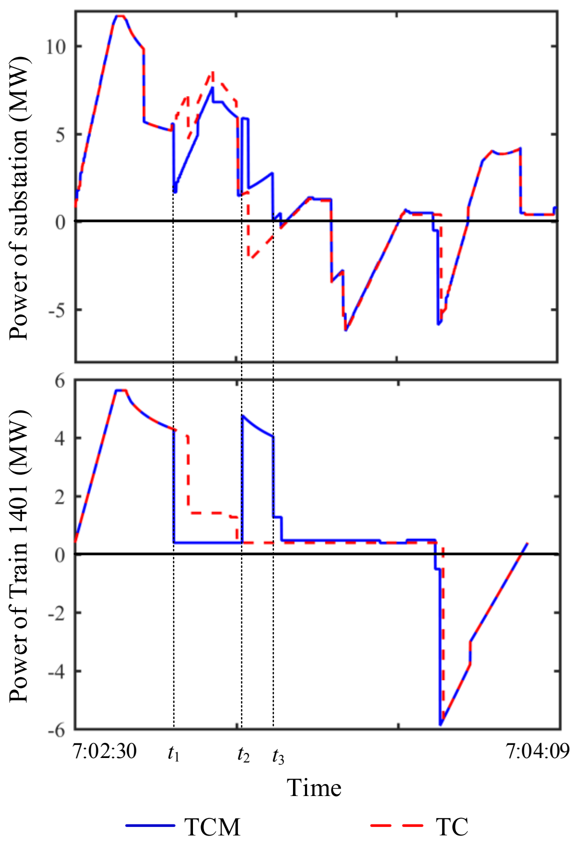

Figure 9. The power flow of time interval [7:02:30 7:04:09]

REFERENCES

[1] A. González-Gil, R. Palacin, P. Batty, and J. P. Powell, “A systems approach to reduce urban rail energy consumption,” *Energy Convers. Manag.*, vol. 80, pp. 509–524, 2014.

[2] X. Yang, X. Li, B. Ning, and T. Tang, “A survey on energy-efficient train operation for urban rail transit,” *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 1, pp. 2–13, 2016.

[3] G. M. Scheepmaker, R. M. P. Goverde, and L. G. Kroon, “Review of energy-efficient train control and timetabling,” *Eur. J. Oper. Res.*, vol. 257, no. 2, pp. 355–376, 2017.

[4] K. Ishikawa, “Application of optimization theory for bounded state variable problems to the operation of trains,” *Bull. JSME*, vol. 11, no. 47, pp. 857–865, 1968.

[5] P. J. Pudney and P. Howlett, “Optimal driving strategies for a train journey with speed limits,” *J. Aust. Math. Soc. Ser. B-Applied Math.*, vol. 36, no. 1, pp. 38–49, 1994.

[6] P. G. Howlett, I. P. Milroy, and P. J. Pudney, “Energy-efficient train control,” *Control Eng. Pract.*, vol. 2, no. 2, pp. 193–200, 1994.

[7] R. Franke, P. Terwiesch, and M. Meyer, “An algorithm for the optimal control of the driving of trains,” *Proc. IEEE Conf. Decis. Control*, vol. 3, pp. 2123–2128, 2000.

[8] J. T. Haahr, D. Pisinger, and M. Sabbaghian, “A dynamic programming approach for optimizing train speed profiles with speed restrictions and passage points,” *Transp. Res. Part B Methodol.*, vol. 99, pp. 167–182, 2017.

[9] B. Jin, P. Sun, M. Xu, and Q. Wang, “Train Timetable and Trajectory Optimization Using Improved State-space MILP,” *Chinese Control Conf. CCC*, pp. 7748–7753, 2018.

[10] C. S. Chang and S. S. Sim, “Optimising train movements through coast control using genetic algorithms,” *IEE Proc. - Electr. Power Appl.*, vol. 144, no. 1, p. 65, 1997.

[11] K. K. Wong and T. K. Ho, “Coast Control of Train Movement with Genetic Algorithm,” *Int. J. Syst. Sci.*, vol. 35, no. 13–14, pp. 835–846, 2005.

[12] I. Hasegawa and S. Uchida, “Braking systems,” *Japan Railway Transp. Rev.*, vol. 20, pp. 52–59, Jun. 1999.

[13] Z. Tian, P. Weston, N. Zhao, S. Hillmansen, C. Roberts, and L. Chen, “System energy optimisation strategies for metros with regeneration,” *Transp. Res. Part C Emerg. Technol.*, vol. 75, pp. 120–135, 2017.

[14] M. Miyatake and H. Ko, “Optimization of train speed profile for minimum energy consumption,” *IEEJ Trans. Electr. Electron. Eng.*, vol. 5, no. 3, pp. 263–269, 2010.

[15] Y. V. Bocharnikov, A. M. Tobias, and C. Roberts, “Reduction of train and net energy consumption using genetic algorithms for trajectory optimisation,” *Proc. IET Conf. Railway Traction Syst.*, pp. 32–36, 2010.

[16] X. Sun, H. Cai, X. Hou, M. Zhang, and H. Dong, “RBE utilization by multi train cooperation,” *2014 17th IEEE Int. Conf. Intell. Transp. Syst. ITSC 2014*, pp. 139–144, 2014.

[17] X. Sun, H. Lu, and H. Dong, “Energy-Efficient Train Control by Multi-Train Dynamic Cooperation,” *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 11, pp. 3114–3121, 2017.

[18] X. Li and H. K. Lo, “Energy minimization in dynamic train scheduling and control for metro rail operations,” *Transp. Res. Part B Methodol.*, vol. 70, pp. 269–284, 2014.

[19] Y. Zhou, Y. Bai, J. Li, B. Mao, and T. Li, “Integrated Optimization on Train Control and Timetable to Minimize Net Energy Consumption of Metro Lines,” *J. Adv. Transp.*, 2018.